Workshop
Topology and Computer 2015

A workshop "Topology and Computer 2015" will be held as follows. This workshop is supported financially by the Grants-in-Aid for Scientific Research (S) (Director: Takashi Tsuboi #24224002) and the Grant-in-Aid Scientific Research (C) (Director: Kazuhiro Ichihara #26400100).

Date: November 6 – 8, 2015
Location: College of Humanities and Sciences, Nihon University
Lecture Hall in 8th Bldg. (Nov. 6), Room 3504 in 3rd Bldg. (Nov. 7),
Oval Hall in the Library (Nov. 8)
Address: 3-25-40 Sakurajosui Setagaya-Ku, Tokyo, Japan
Web: http://auemath.aichi-edu.ac.jp/˜ainoue/workshop/TopologyComputer2015.html

Program

November 6

13:40 – 13:50 Opening

13:50 – 14:15 Ryosuke Yamazaki (The University of Tokyo)
Jørgensen numbers of some Kleinian groups
on the boundary of the classical Schottky space

14:25 – 14:50 Kazue Okubo (Nara Women's University)
Discreteness of some four generator Möbius groups

15:05 – 15:55 Hidetoshi Masai (The University of Tokyo)
Experiments on random braids

16:10 – 17:00 Kaname Matsue
(The Institute of Statistical Mathematics, Coop-with-Math Program (MEXT))
Rigorous numerics for fast-slow systems – Topological shadowing approach –

November 7

10:00 – 10:25 Taira Akiyama and Ayaka Shimizu (Gunma National College of Technology)
Warping incidence matrix and Reidemeister moves

10:35 – 11:00 Yuta Nozaki (The University of Tokyo)
Knots in RP^3 and their lifts to S^3

11:15 – 12:05 Mark Bell (University of Illinois)
Flip graphs and encodings of mapping classes

12:05 – 13:30 Lunch

13:30 – 13:55 Ayaka Oigo (Nara Women's University)
A computer experiment on pseudomodular groups

14:10 – 15:00 Yoshio Okamoto (The University of Electro-Communications)
Non-planar graph drawing

15:15 – 16:05 Mark Bell (University of Illinois)
The conjugacy problem for mapping class groups

16:20 – 17:10 Kazushige Terui (RIMS)
Can a computer prove mathematical theorems?
— from the viewpoint of foundations of mathematics and computer science

November 8

10:00 – 10:25 Yuumu Rikiishi (Meiji University)
On a system which allows us to simulate smoothing operation on knot projections using dynamics of spring (joint work with Kazushi Ahara)

10:35 – 11:25 Kento Nakamura (Meiji University)
Visualization of Kleinian groups: fractal artworks and rendering techniques

11:40 – 12:30 Hirokazu Shimauchi (Yamanashi Eiwa College)
A numerical algorithm for quasiconformal mappings (joint work with R. Michael Porter)

12:30 – 12:40 Closing

Organizers: Ayumu Inoue (Aichi Univ. of Education), Kazuhiro Ichihara (Nihon Univ.), Fumikazu Nagasato (Meijo Univ.), Ayako Ido (Aichi Univ. of Education)

Abstracts

Ryosuke Yamazaki (The University of Tokyo)
Jørgensen numbers of some Kleinian groups on the boundary of the classical Schottky space

It is well-known that Jørgensen's inequality gives a necessary condition for a two-generator non-elementary subgroup of the Möbius transformation group to be discrete (i.e., Kleinian group). Oichi-Sato showed that for every real number $r > 4$, there is a classical Schottky group Γ such that the Jørgensen number of Γ is equal to r. In this talk, I will introduce my computational result of Jørgensen numbers of some Kleinian groups, and give an extension of Oichi-Sato's theorem.

Kazue Okubo (Nara Women's University)
Discreteness of some four generator Möbius groups

The $\mathrm{SL}(2, \mathbb{C})$-character variety of the free group of rank two is identified with the complex vector space of dimension three. A subset this character variety called the Maskit slice is well studied in various contexts. In this talk, we study some four generator Möbius groups constructed from two generator Möbius groups in the Maskit slice using the method of plumbing. In this talk, we will present our computer experiments on the discreteness of these groups using the limit sets, the existence of elliptic elements, and Jørgensen's inequality.

Hidetoshi Masai (The University of Tokyo)
Experiments on random braids

We consider the simple random walks on braid groups. By the work of Maher, it is known that such a random walk gives rise to pseudo-Anosov elements with probability exponentially converging to 1. But how "quick" do they converge? More precisely, for the braid group of degree n, how many steps do we need to have pseudo-Anosov elements? In this talk, we report computer experiments for this question. We also consider the number of components of the closure, degree of Alexander polynomial, and the hyperbolic volume of the complement of the closure of random braids.

Kaname Matsue
(The Institute of Statistical Mathematics, Coop-with-Math Program (MEXT))
Rigorous numerics for fast-slow systems – Topological shadowing approach –

We provide a rigorous numerical computation method to validate periodic, homoclinic and heteroclinic orbits as the continuation of singular limit orbits for the fast-slow system $x' = f(x, y, \varepsilon), y' = \varepsilon g(x, y, \varepsilon)$. Our validation procedure is based on topological tools called isolating blocks, cone condition and covering relations. Such tools provide us with existence theorems of global orbits which shadow singular orbits in terms of a new concept, the covering-exchange. Additional topological techniques called "slow shadowing" and "m-cones" are also developed. These techniques give us not only generalized topological verification theorems, but also easy implementations for validating trajectories near slow manifolds in a wide range, via rigorous numerics. Our procedure is available to validate global orbits not only for sufficiently small ε but all ε in an explicitly given half-open interval $(0, \varepsilon_0]$.

Taira Akiyama and Ayaka Shimizu (Gunma National College of Technology)
Warping incidence matrix and Reidemeister moves

We introduce the warping incidence matrix, a new description for knots. In this talk we study the behavior of the matrix under Reidemeister moves, and discuss the sequence of Reidemeister moves between a monotone diagram and the trivial diagram.

Yuta Nozaki (The University of Tokyo)
Knots in RP^3 and their lifts to S^3

Matveev proposed a question: Do there exist non-equivalent knots in RP^3 such that their lifts to S^3 are equivalent knots? In this talk, we discuss some related problems and give a partial answer to the question.

Mark Bell (University of Illinois)
Flip graphs and encodings of mapping classes

One of the first steps in tacking a problem in computational topology is to express your topological problem as a combinatorial one – suitable for computers. We will look at a such a technique for mapping classes, the isotopy classes of maps of a surface. We will do this by considering a space which these mapping classes have a natural action on, namely the flip graph. This graph comes from the many different ways of triangulating the surface and has many useful properties that make it suitable for these sorts of calculations such as being locally finite. We will look at some techniques for efficiently finding paths through this graph, allowing us to quickly construct the representation of a given mapping classes. This construction has been implemented and is available as part of the Python package flipper.

Ayaka Oigo (Nara Women's University)
A computer experiment on pseudomodular groups

Let G be a discrete subgroup of $\mathrm{PSL}(2, \mathbb{R})$. The set of parabolic fixed points of G is called the cusp set. It is well known that the cusp set of the modular group ($\mathrm{PSL}(2, \mathbb{Z})$) coincides with the set of rational numbers and $1/0$. Long and Reid gave examples of discrete groups, whose cusp sets are equal to the set of rational numbers and $1/0$, which are not commensurable to the modular group. These examples are called "pseudomodular groups". In this talk, we will present our computer experiments trying to give new examples of pseudomodular groups.

Yoshio Okamoto (The University of Electro-Communications)
Non-planar graph drawing

Recent research trend on non-planar graph drawing is discussed. The topics include k-planarity, right-angle-crossing drawing, and slope numbers. Several open problems in the literature will also be presented.

Mark Bell (University of Illinois)
The conjugacy problem for mapping class groups

We will discuss a new approach for solving the conjugacy problem for mapping class groups. This solution uses the action of mapping classes, again encoded via paths in the flip graph, on a second space – the space of measured laminations. Points in this space, which generalises the space of curves on the surface, can also be efficiently described using triangulations. A deep theorem of Nielsen and Thurston says that the action of a given mapping class on this space has a (projective) fixed point. From these fixed points one can compute enough combinatorial properties to uniquely determine the conjugacy class of a given mapping class and so solve the conjugacy problem for mapping class groups. This construction also has several connections to veering triangulations of fibred 3-manifolds and, unlike techniques in braid groups using Garside structures, this algorithm is effective and is also included as part of the Python package flipper.

Kazushige Terui (RIMS)
Can a computer prove mathematical theorems?
— from the viewpoint of foundations of mathematics and computer science

This talk is an informal discussion on the possibility and limitation of automated theorem proving in mathematics. While it is undoubtedly true that the core of mathematics lies in creative activities such as making a conjecture and proposing a new concept, we can for a while restrict our attention to a less creative activity of proving a target theorem from a given set of axioms via formal logical reasoning. In this "proof game", can a computer compete with a human mathematician? — that is the question.

Continuing development of automated theorem proving apparently suggests a future (partial) success in that direction. But is it really the case? Meanwhile, the traditional theory of foundations of mathematics insists that there is a certain theoretical limitation. But does it really conform to the realistic situation? Finally in foundations of theoretical computer science, "proofs" are often identified with "programs", which are the direct target of intensive study. Can it be applied to our goal of computer-automated mathematics?

After briefly reviewing these issues, we will discuss, from a *personal* point of view, what would be a potential approach to automated theorem proving in mathematics — if it is possible by any chance.

Yuumu Rikiishi (Meiji University)
On a system which allows us to simulate smoothing operation on knot projections using dynamics of spring (joint work with Kazushi Ahara)

In this study, we present a system which allows us to obtain 'good' figures of knot projections in the plane, using 'double spring dynamics'. The user can input a knot figure by mouse dragging, and this system allows us to simulate smoothing and un-smoothing operation on the screen.

Kento Nakamura (Meiji University)
Visualization of Kleinian groups: fractal artworks and rendering techniques

The limit set of Kleinian groups has a fractal structure in most cases. The image artworks of the limit sets are complicated and beautiful, and they feast our eyes on wonders. Other than Kleinian fractals, there are a wide variety of fractals such as Mandelbrot set and Julia set. Many lovers and researchers of fractals established big communities on the web, and they study optimization of rendering fractal images and show many kinds of technique of rendering. In this talk, we introduce the recent works (artworks and software) of the speaker as well as topics of the communities and of technique of fractal rendering.

Hirokazu Shimauchi (Yamanashi Eiwa College)
A numerical algorithm for quasiconformal mappings (joint work with R. Michael Porter)

We will present a simple algorithm for quasiconformal mappings of planar disk. The disk is triangulated in a simple way and a quasiconformal mapping is approximated by piecewise linear mappings. The sequence of the approximation converges to the true solution at least if its Beltrami coefficient is continuously differentiable. We will also show several numerical experiments. This talk is based on a joint work with R. Michael Porter (CINVESTAV).